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Experimental study of dispersion and modulational instability of surface gravity waves on constant vorticity currents

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Experimental Study of Disp
Modulational Instability of Su
Waves on Constant Vortici

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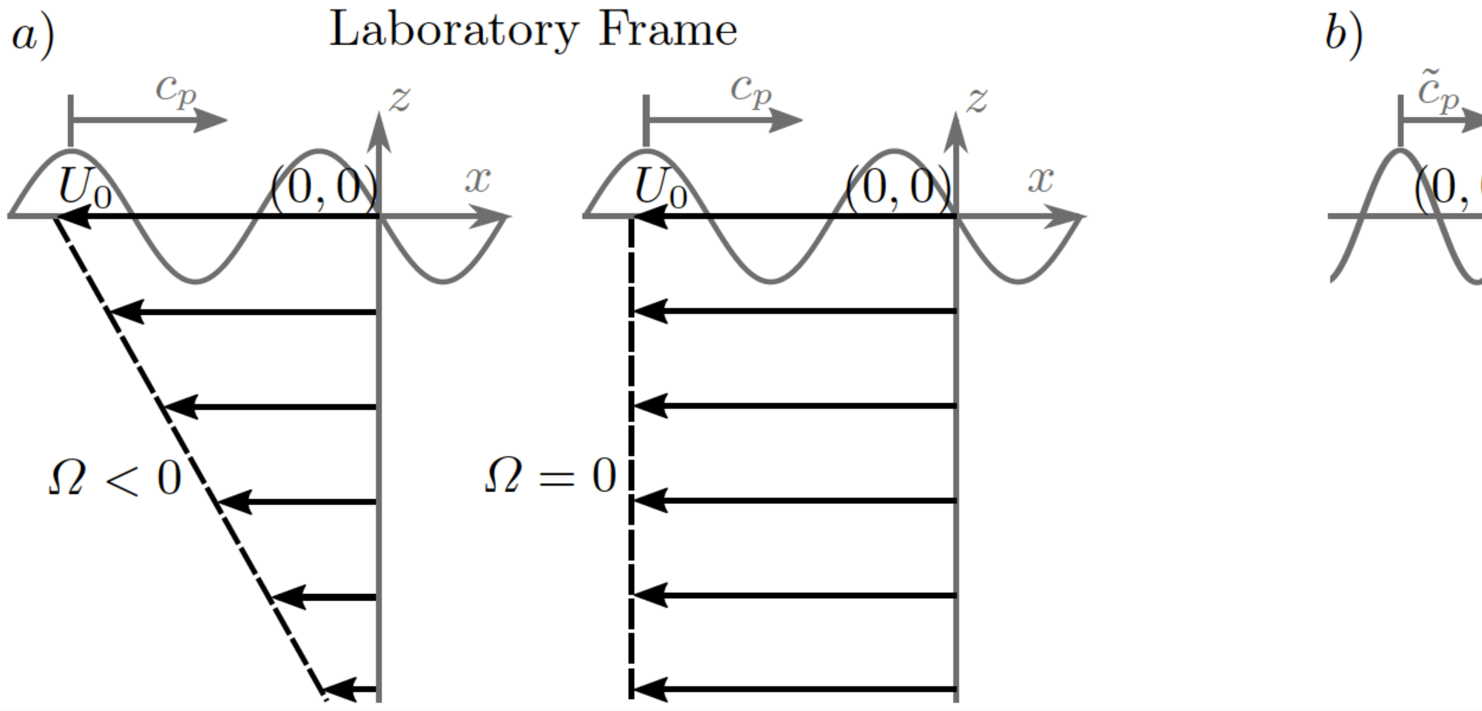
Experiments for negatively sheared current



Linear background current: $U = U_0 + \Omega z.$

Tilde denotes surface current reference frame: $\omega = \tilde{\omega} +$

Waves have potential: $\boldsymbol{u} = U(z)\hat{\boldsymbol{i}} + \boldsymbol{\nabla}\phi,$

Governing equations and boundary condition

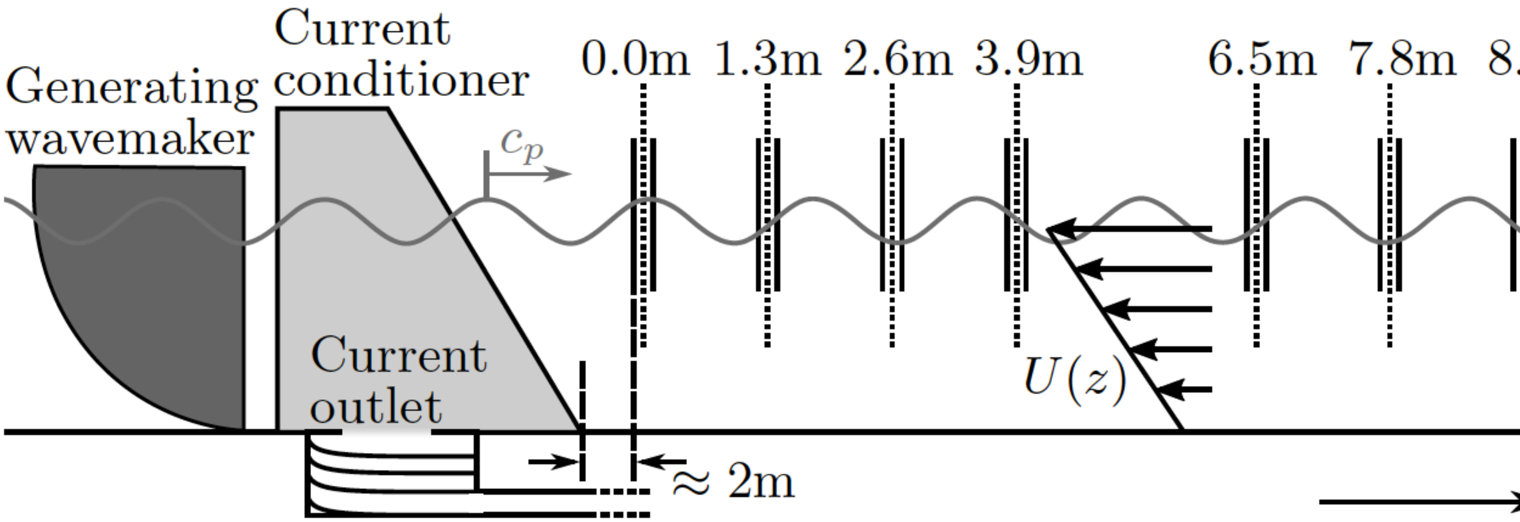
Laplace: $\nabla^2\phi = 0 \qquad -d < z < \eta(x,t)$

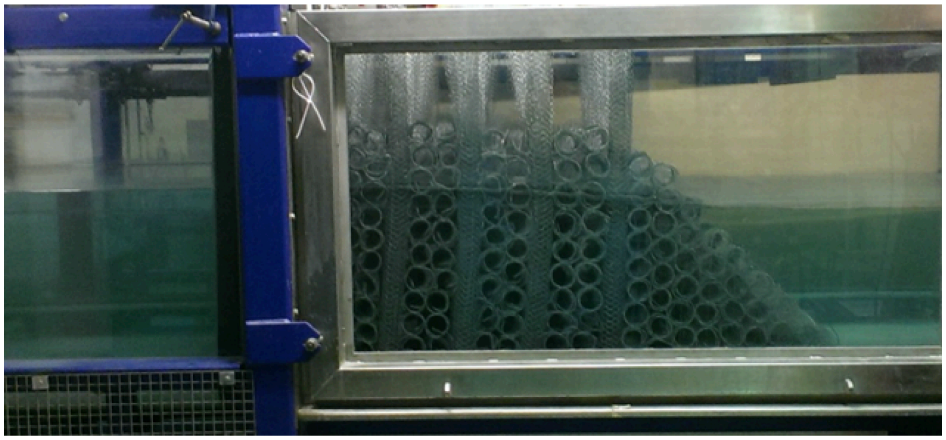
Kinematic free surface boundary condition: $\eta_t + (\Phi_x + \Omega\eta)\eta_x -$

Dynamic free surface boundary condition: $\Phi_t + \frac{1}{2}\Phi_x^2 + \frac{1}{2}\Phi_z^2 + \Omega$

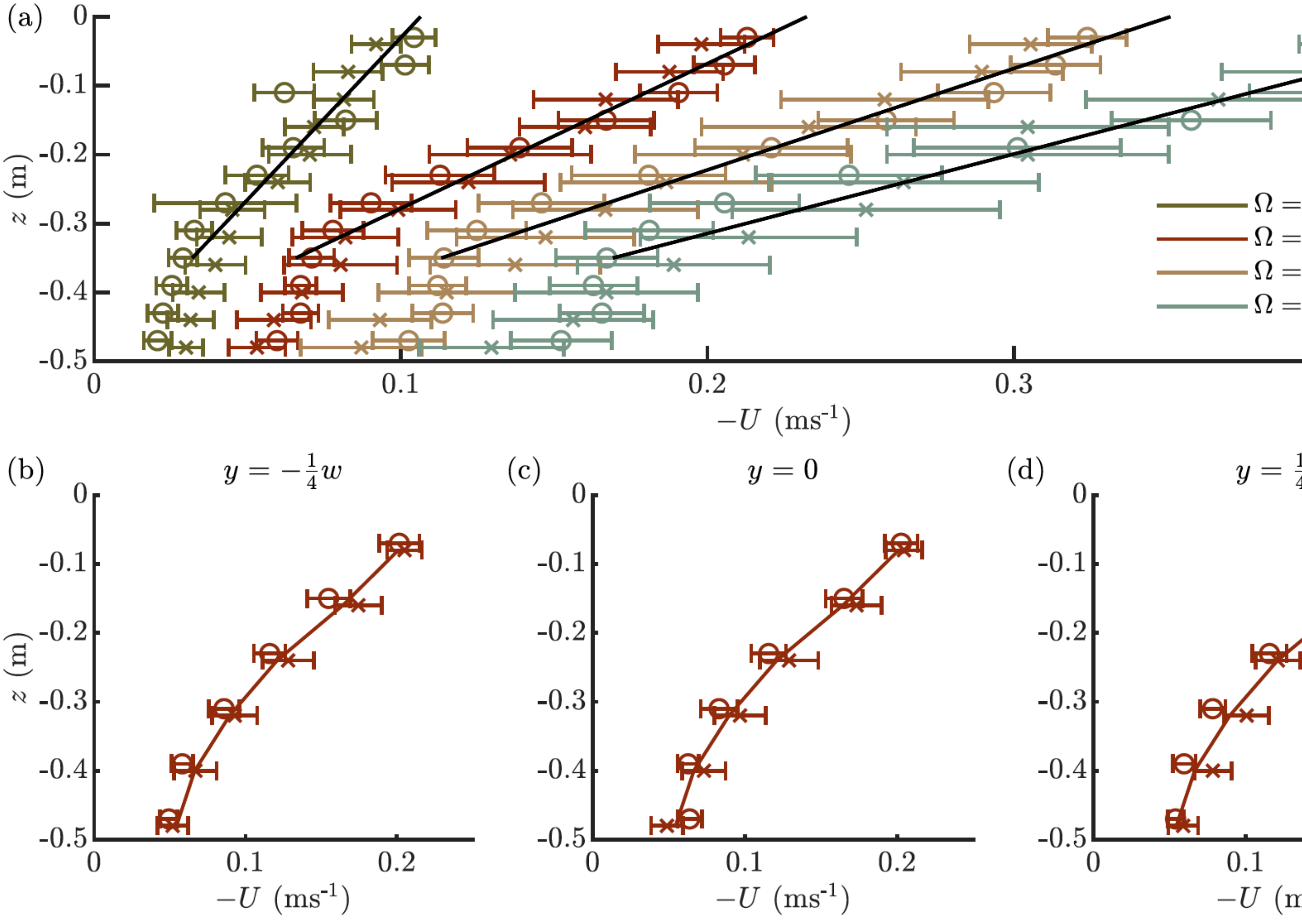
Free surface values: $\Psi \equiv \psi(z = \eta(x,t)) \qquad \Phi \equiv \phi(z$

Laboratory experiments (UCL)

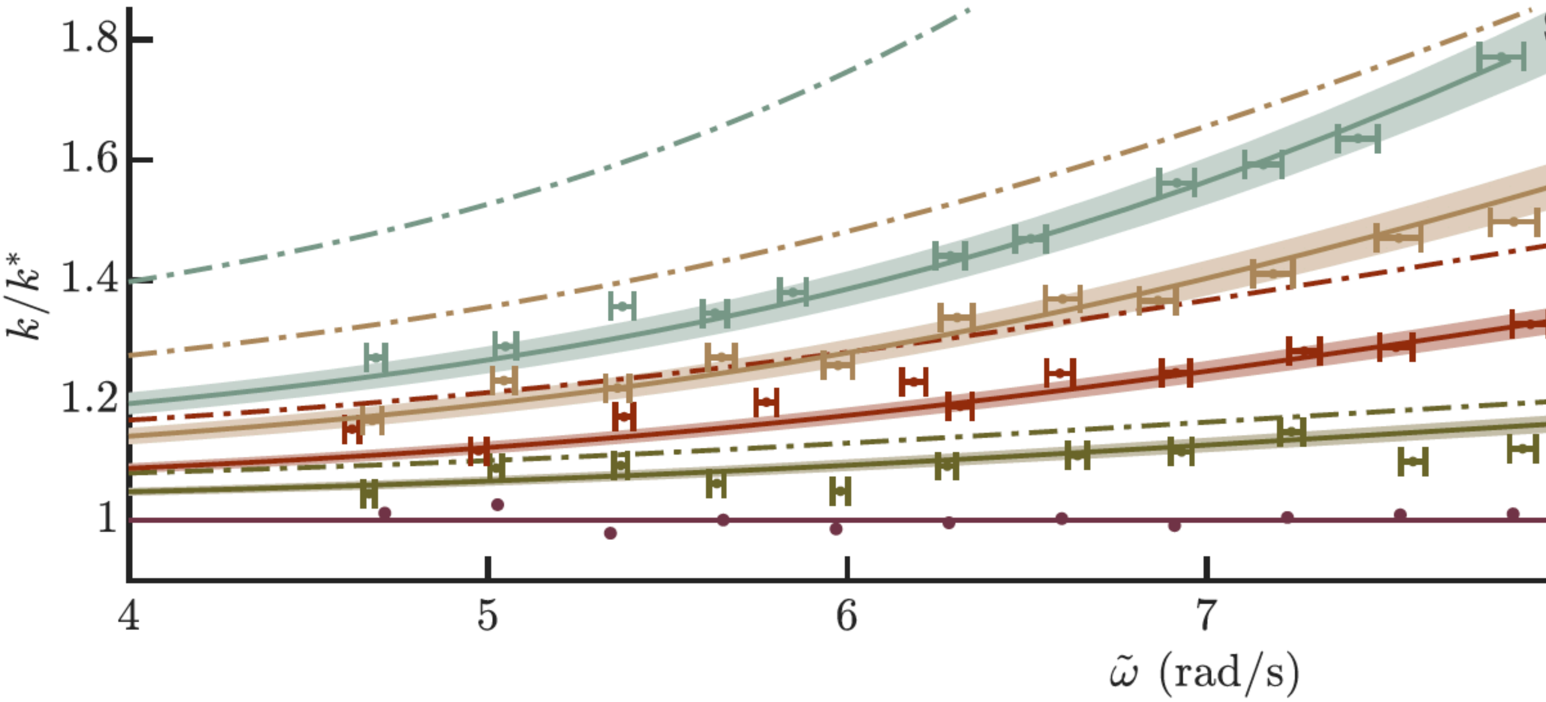




Velocity profiles



Linear dispersion relationship: $\tilde{\omega}_0^2 + (\tilde{\omega}_0 \mathit{\Omega} - gk$



Vor-NLSE

Scaled space and time: $\xi = \epsilon(\tilde{x} - \tilde{c}_g t) \qquad \tau = \epsilon^2 t$

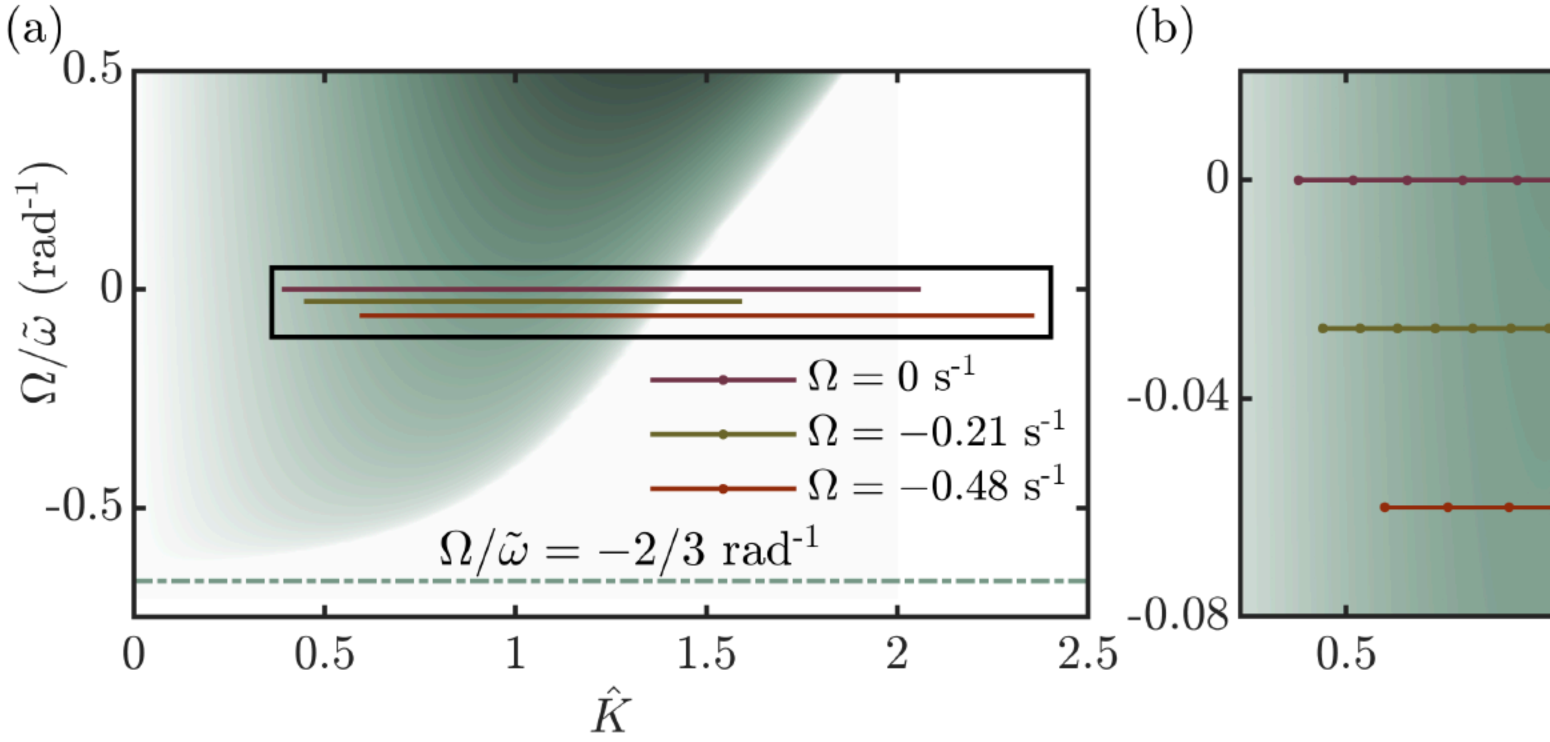
NLSE: $iA_\tau + LA_{\xi\xi} - M|A|^2 A = 0.$

Coefficients: $L = -\dfrac{\tilde{\omega}_0(1+\bar{\Omega})^2}{k_0^2(2+\bar{\Omega})^3}$ and $M = \dfrac{\tilde{\omega}_0 k_0^2}{8(1+\bar{\Omega})}$

$$\bar{\Omega} = \Omega/\tilde{\omega}_0$$

From envelope to free surface: $\eta^{(1)} = \mathrm{Re}\Big[\epsilon A(\xi,\tau) e^{i(k_0\tilde{x}-}$

Linear stability analysis

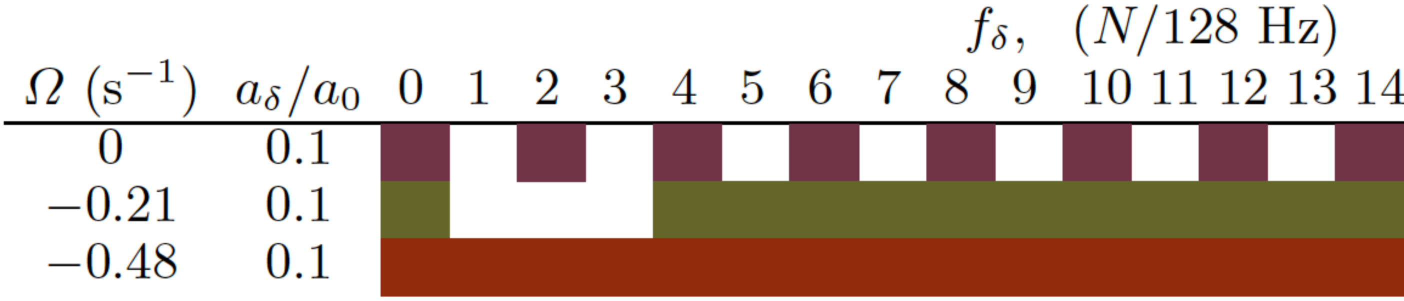


$$A = [a_0 + \delta(\tau, \xi)]e^{-iMa_0^2\tau}$$

$$\tilde{\gamma} = \pm\sqrt{K^2L(K}$$

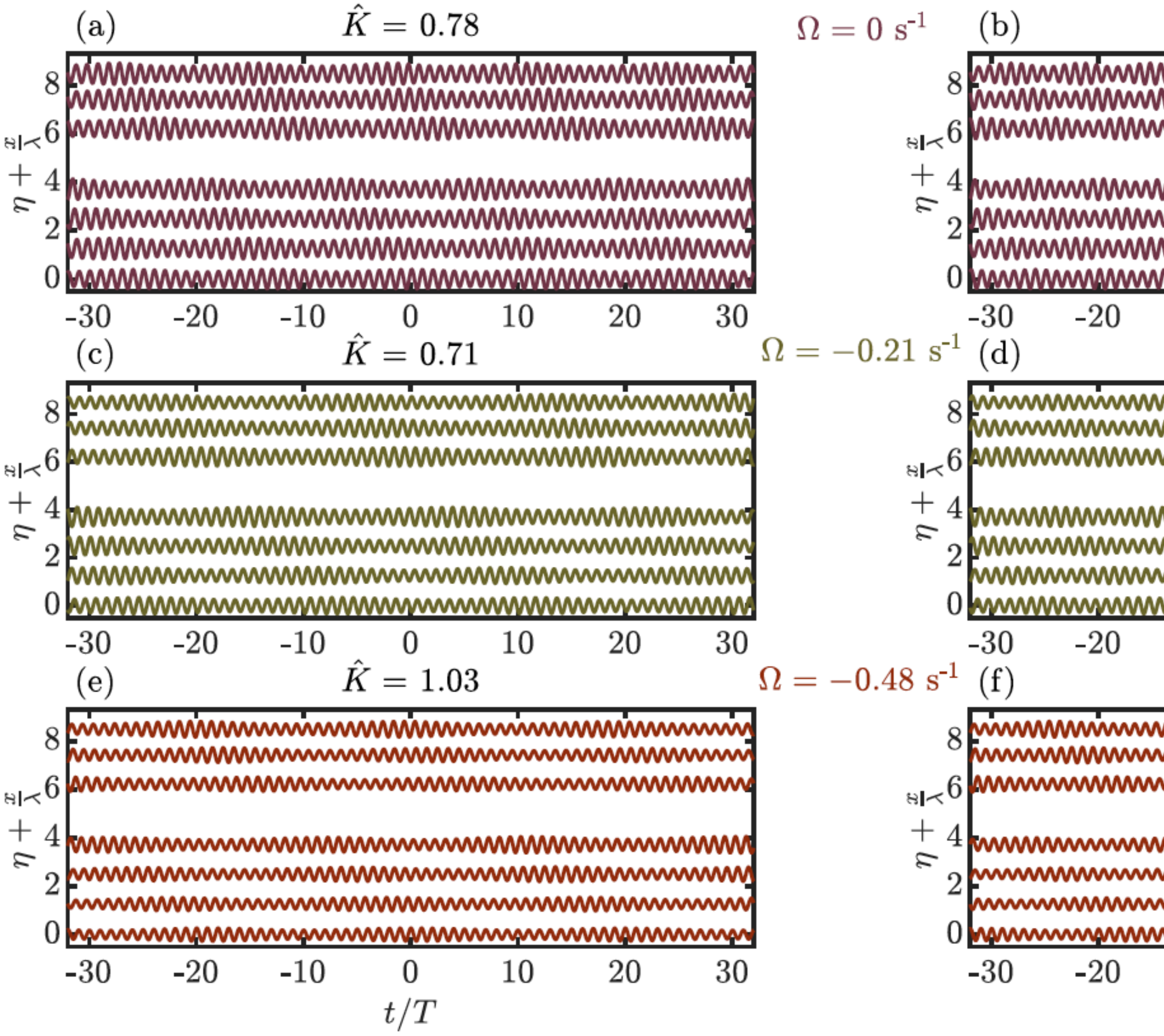
Matrix of experiments

Ω (s^{-1})	ω ($rad\,s^{-1}$)	ka_0
0	7.62	0.15
−0.21	7.17	0.12
−0.48	6.63	0.10

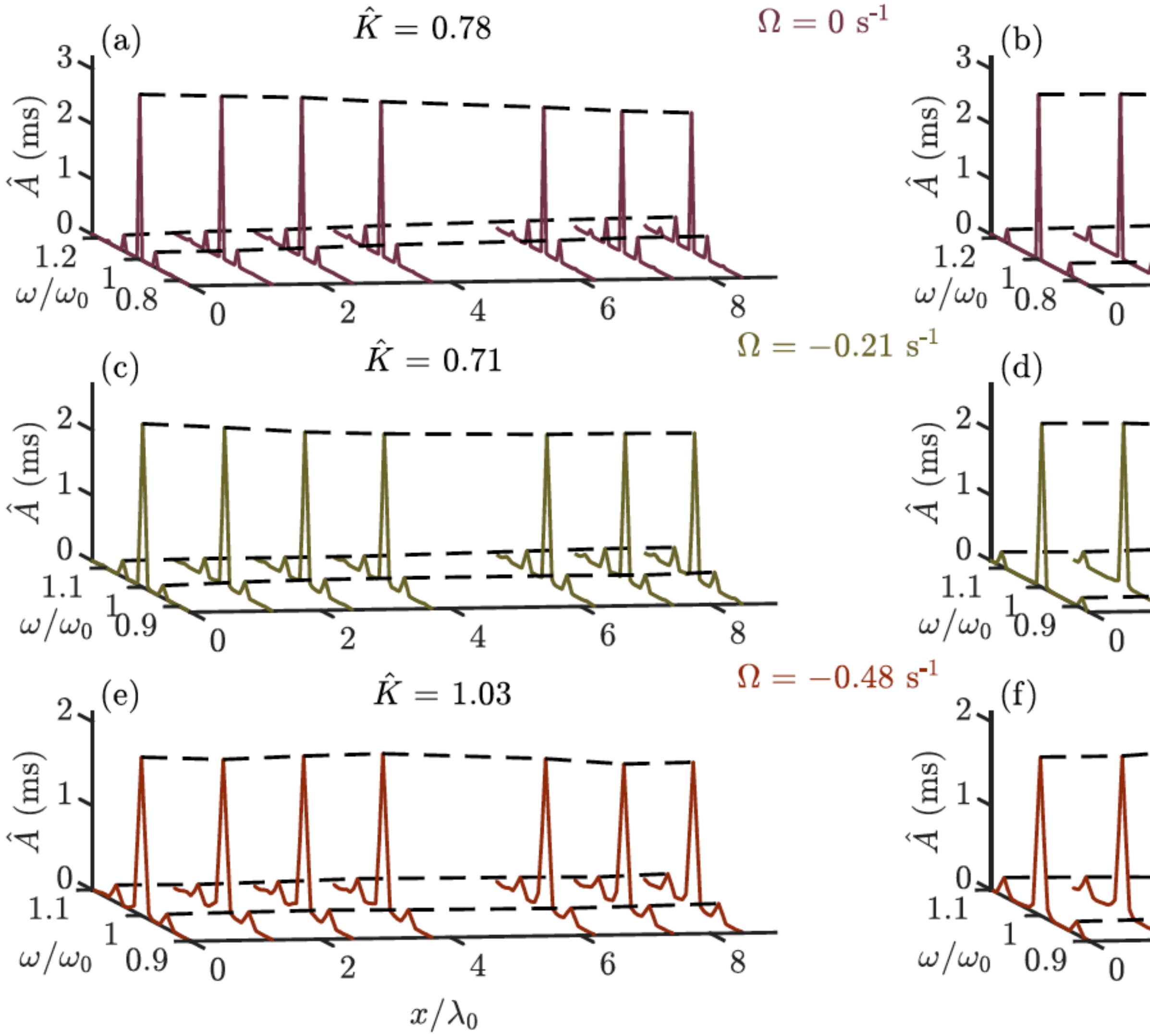


		f_δ, ($N/128$ Hz)														
Ω (s^{-1})	a_δ/a_0	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
0	0.1	🟪		🟪		🟪		🟪		🟪		🟪		🟪		🟪
−0.21	0.1	🟩				🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩
−0.48	0.1	🟥	🟥	🟥	🟥	🟥	🟥	🟥	🟥	🟥	🟥	🟥	🟥	🟥	🟥	🟥

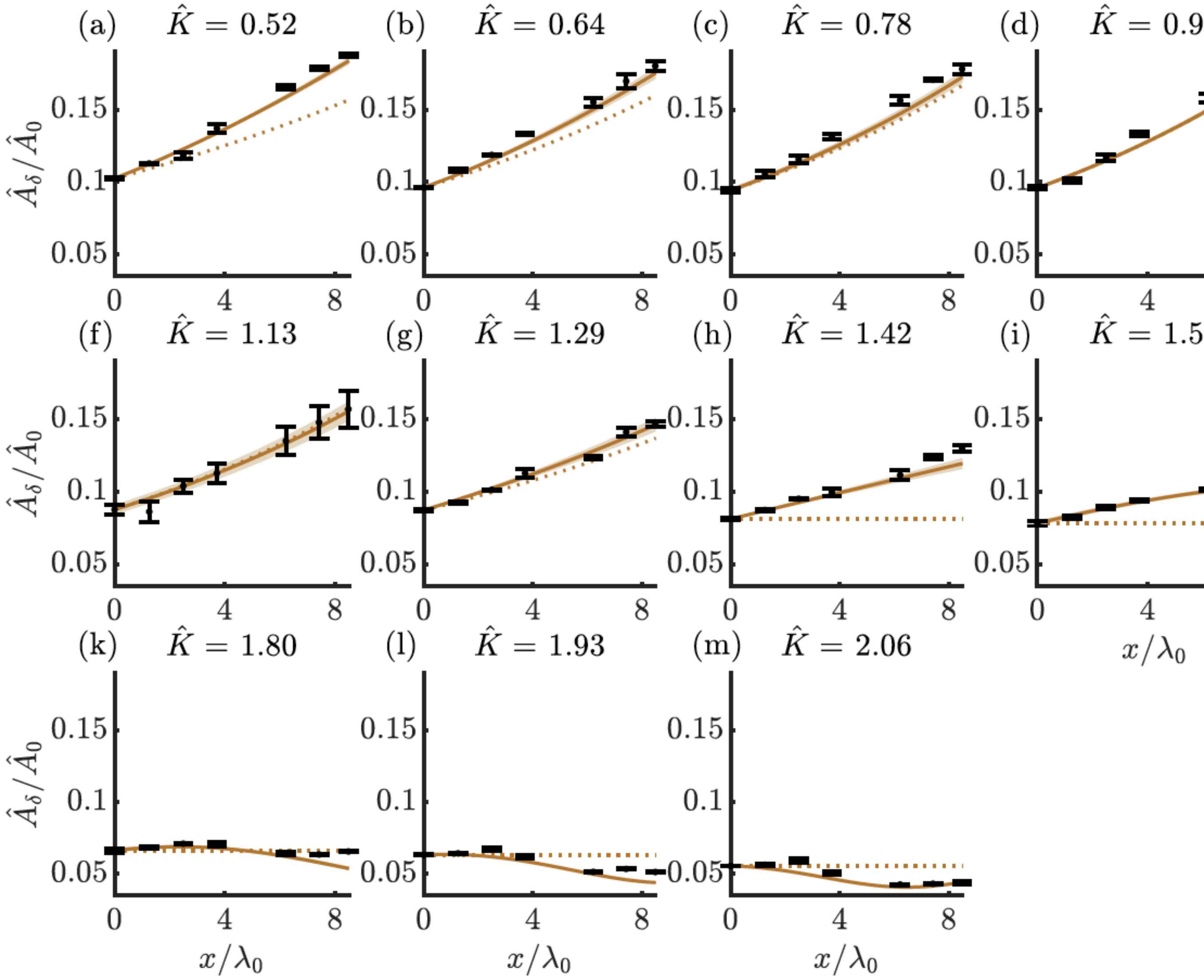
Example time series

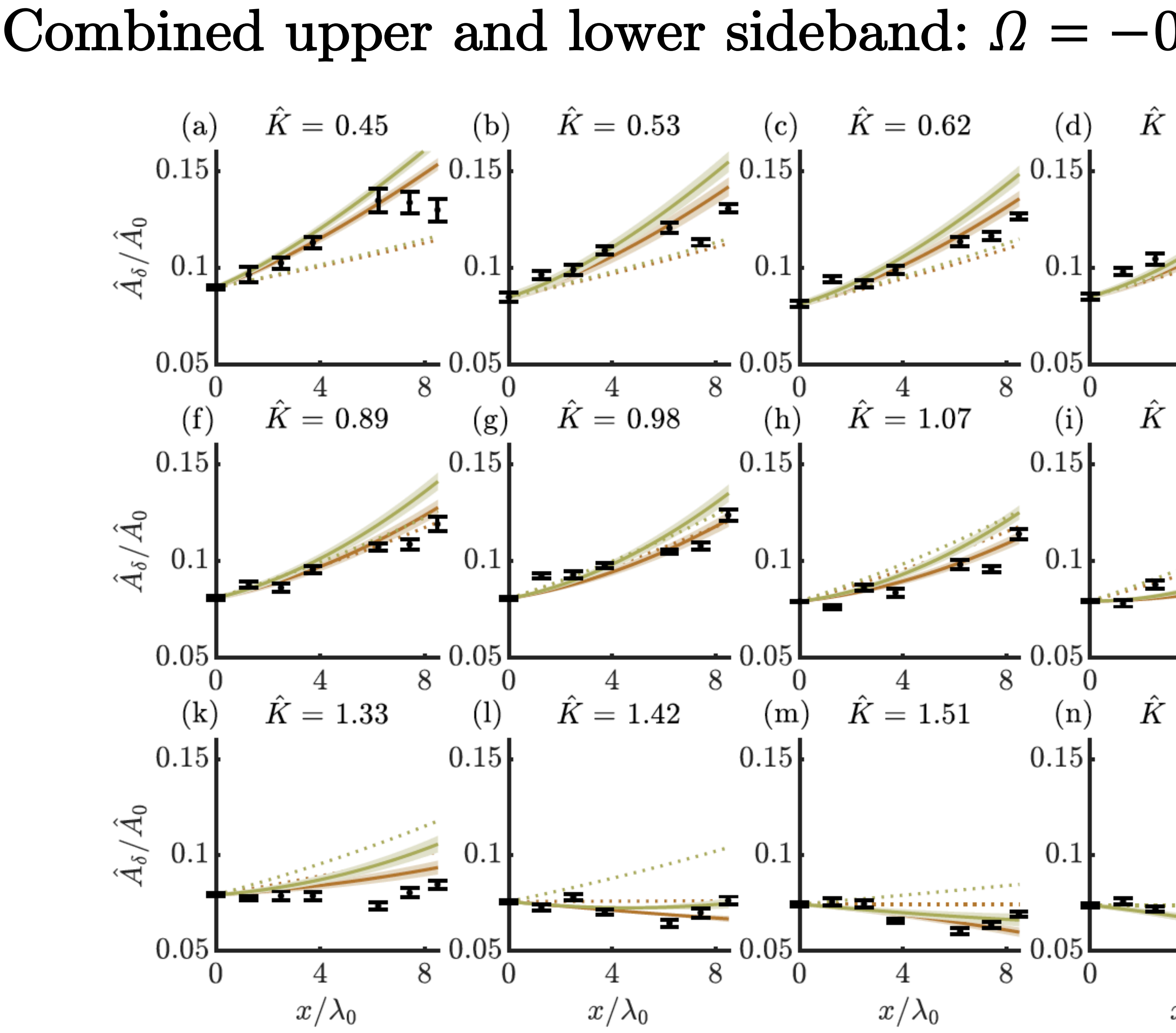


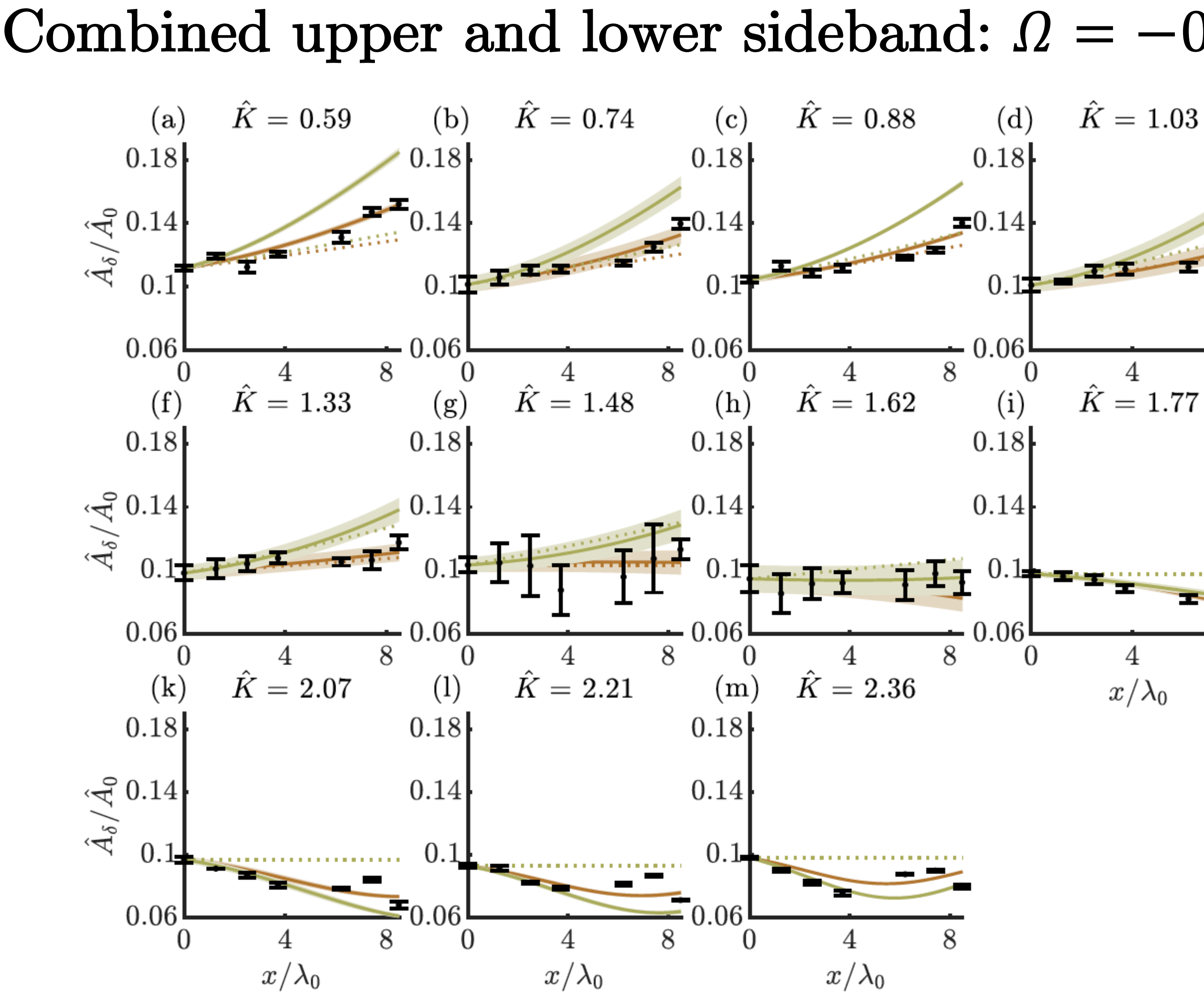
Example time spectra



Combined upper and lower sideband: $\mathbf{\Omega = 0}$




Combined upper and lower sideband: $\Omega = -0$
(a) $\hat{K} = 0.45$
(b) $\hat{K} = 0.53$
(c) $\hat{K} = 0.62$
(d) $\hat{K}$
(f) $\hat{K} = 0.89$
(g) $\hat{K} = 0.98$
(h) $\hat{K} = 1.07$
(i) $\hat{K}$
(k) $\hat{K} = 1.33$
(l) $\hat{K} = 1.42$
(m) $\hat{K} = 1.51$
(n) $\hat{K}$
$\hat{A}_\delta / \hat{A}_0$
0.15
0.1
0.05
0
4
8
x/λ_0


Combined upper and lower sideband: $\Omega = -0$
(a) $\hat{K} = 0.59$
(b) $\hat{K} = 0.74$
(c) $\hat{K} = 0.88$
(d) $\hat{K} = 1.03$
(f) $\hat{K} = 1.33$
(g) $\hat{K} = 1.48$
(h) $\hat{K} = 1.62$
(i) $\hat{K} = 1.77$
(k) $\hat{K} = 2.07$
(l) $\hat{K} = 2.21$
(m) $\hat{K} = 2.36$
$\hat{A}_\delta/\hat{A}_0$
0.18
0.14
0.1
0.06
0
4
8
x/λ_0

Maximum amplification

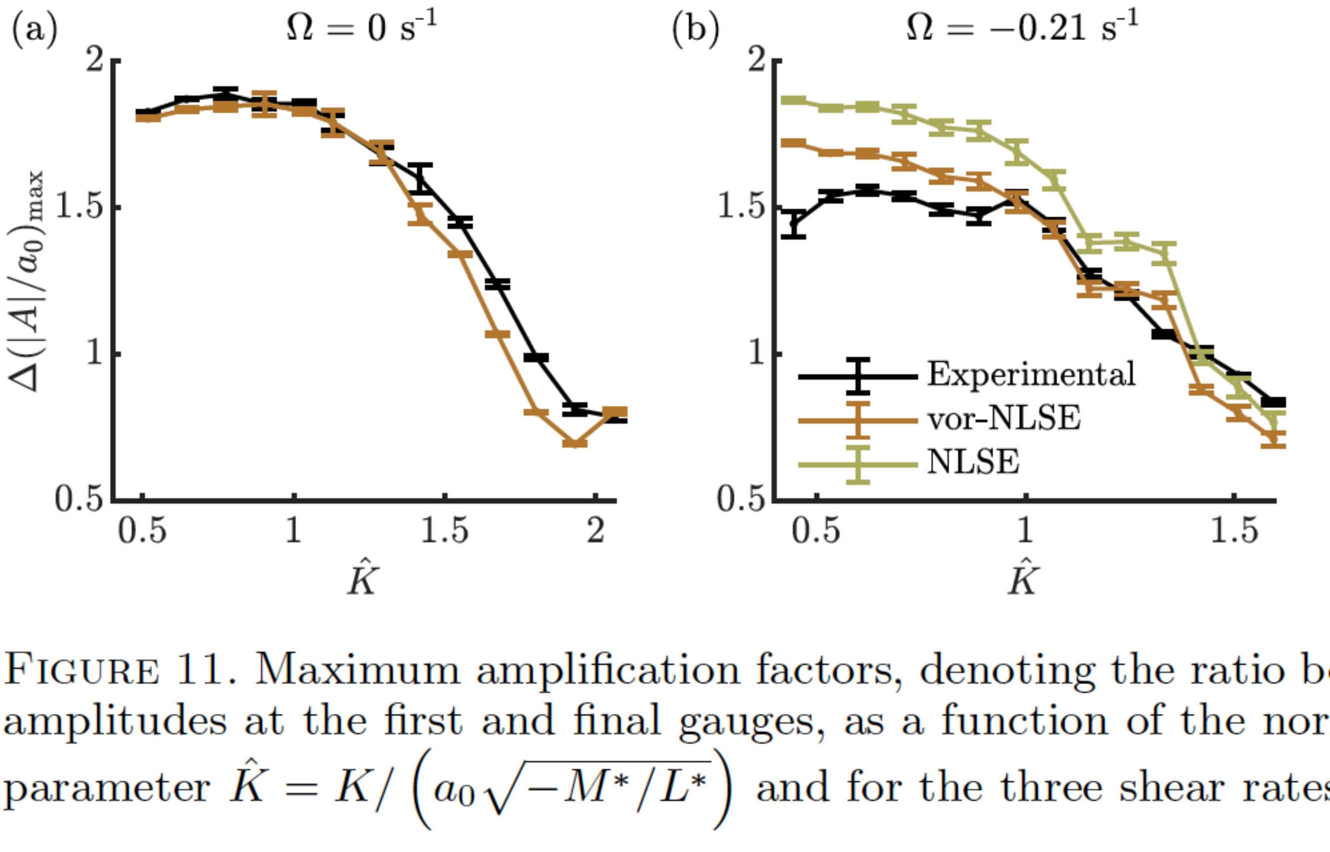


FIGURE 11. Maximum amplification factors, denoting the ratio be amplitudes at the first and final gauges, as a function of the nor parameter $\hat{K} = K/\left(a_0\sqrt{-M^*/L^*}\right)$ and for the three shear rates

Conclusions

- Can robustly observed shear-modified linear dispersion relationship
  currents).
- Negative shear stabilizes the modulational instability: vor-NLSE be

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